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A tangled cosmological magnetic field in the early universe creates Alfvén-wave modes that induce small rotational velocity perturbations on the last scattering surface of the microwave background radiation. We compute the contribution of these vector modes to the small angular scale CMBR polarization anisotropy. We show that a magnetic field which redshifts to a present value of $3\times10^{-9}$ Gauss produces polarization anisotropies of about $0.2\mu K$ to $0.8\mu K$ at $l$ values of around 300 to 1500, or angular scales of 10 arc-minutes or smaller. Further, the signal is dominated by the odd parity, B-type polarization. This feature distinguishes signals due to magnetic field perturbations from those due to, say inflation generated, scalar ones and could help in their detection.

PACS Numbers : 98.62.En, 98.70.Vc, 98.80.Cq, 98.80.Hw

Magnetic fields in astronomical objects, like galaxies, could grow by the amplification of small seed fields by turbulent dynamo action [1]. However, the need to produce magnetic helicity in galaxies seems to severely constrain the efficiency of such dynamo action [2]. Alternatively, the galactic field could be a remnant of a cosmological field of primordial origin [3], although, as yet, there is no entirely compelling mechanism for producing the required field [4]. A primordial field that expanded to contribute a present field strength of order $10^{-9}$ Gauss, tangled on galactic scales, could also impact significantly on galaxy formation [5,6]. It is of considerable interest, therefore, to find different ways of constraining or detecting such primordial fields [7]. Observations of anisotropies in the cosmic microwave background radiation (CMBR), provide a potentially powerful constraint on such fields. Indeed, in earlier work it was found that the CMBR temperature isotropy can be used to place limits, of order several nano-Gauss, on both the uniform [8] and tangled components of the magnetic fields [9,10]. However, temperature anisotropies in popular inflation inspired models of structure formation, are expected to be dominated by scalar contributions due to perturbations other than those induced by magnetic fields. So the detection of a magnetic field induced signal is likely to be difficult, except possibly on scales smaller than the Silk damping scale [9]. Here we point out the advantage of using alternatively, the polarization anisotropy to constrain primordial magnetic fields. Note that scalar perturbations only produce, what is known as E-type polarization anisotropy. On the other hand, as we show here, tangled magnetic fields which drive significant vector perturbations, will also lead to a distinctive, significant and potentially detectable B-type polarization anisotropy of the CMBR. This could help in separating their contribution from scalar contributions, to detect/constrain such tangled fields.

Polarization of the CMBR arises from the Thomson scattering of radiation from free electrons, and is sourced by the quadrupole component of the CMBR anisotropy. The equation governing the evolution of temperature and polarization anisotropy for vector perturbations have been derived in great detail in Ref. [11], in the total angular momentum representation. We will use their results extensively below. The anisotropy in the temperature and polarization is expanded in terms of tensor spherical harmonics. This enables one to write evolution equations, for the moments, $\Theta_l^{(m)}$, $E_l^{(m)}$ and $B_l^{(m)}$, of the temperature anisotropy ($\Delta T/T$), the electric (E-) type and the odd parity, magnetic (B-) type polarization anisotropies, respectively. Here $l$ stands for the multipole number and $m = 0, \pm1, \pm2$, respectively, for scalar, vector and tensor perturbations. For vector perturbations ($m = \pm1$), it can be shown [11] that the magnetic type contribution dominates the polarization anisotropy. Its evolution is given by (Eqn. (77), (62) and (18) of [11] ),

$$\frac{B_l^{(m)}(\tau_0,k)}{2l+1} = -\sqrt{6}\int_0^{\tau_0} d\tau g(\tau_0,\tau)P^{(m)}\beta_l^{(m)}(k(\tau_0-\tau)) \tag{1}$$

where $P^{(m)}(k,\tau) = [\Theta_2^{(m)} - \sqrt{6}E_2^{(m)}]/10$ and $\beta_l^{(1)}(x) = \sqrt{(l-1)(l+2)}j_l(x)/2x$, with $j_l(x)$ the spherical Bessel function of order $l$. The 'visibility function', $g(\tau_0,\tau)$, determines the probability that a photon reaches us at the conformal time $\tau_0$ if it was last scattered at the epoch $\tau$. It is given by $g(\tau,\tau') = \dot\kappa(\tau')\exp[-\int_{\tau'}^{\tau}\dot\kappa(\tau'')d\tau'']$, where $\dot\kappa(\tau) = n_e(\tau)\sigma_T a(\tau)$, $n_e$ is the electron number density, $\sigma_T$ is the Thomson cross section, and $a(\tau)$ is the cosmological scale factor normalised to unity at the present. We assume a flat universe throughout.

For standard recombination physics, $g$ is peaked about a small range of conformal times around the time of recombination. We therefore need to calculate the source term $P^{(1)}$, and hence the quadrupole anisotropies around this epoch of last scattering. These can be analytically estimated using the approximation that the departures from tight-coupling between the baryon and the photon fluid is small, or $k/\dot\kappa = kL_\gamma \ll 1$. Here $L_\gamma(\tau) = (\dot\kappa)^{-1}$ is the co-moving, photon mean free path. First, to leading order in the tight coupling approximation, we have zero quadrupoles, and a dipole $\Theta_1^{(1)} = v_B^{(1)}$, where $v_B^{(1)}$ is the magnitude of the (vector component of) baryon fluid velocity field, in Fourier space. However, to the next order the quadrupole is not zero. It is generated from the dipole at the "last but one" scattering of the CMBR. Using the moments of the Boltzmann equations for the temperature and polarization anisotropies (Eq. (60), (63) and (64) of [11]), one gets $\Theta_2^{(1)} = -4E_2^{(1)}/\sqrt{6} = 4kL_\gamma v_B^{(1)}/(3\sqrt{3})$ and hence $P^{(1)} = \Theta_2^{(1)}/4 = kL_\gamma v_B^{(1)}/(3\sqrt{3})$. Using this in Eq. (1) gives an estimate of $B_l^{(1)}$. It is however conventional to quote the results in terms of the angular power spectra $C_l^{BB}$ due to B-type polarization anisotropy. We use Eq. (56) of [11] to relate $C_l^{BB}$ interms of $B_l^{(1)}$ and get

$$C_l^{BB} = 2\frac{(l-1)(l+2)}{l(l+1)}\frac{4\pi}{9}\int_0^\infty \frac{k^2 dk}{2\pi^2}\frac{l(l+1)}{2} \times$$
$$< \left| \int_0^{\tau_0} d\tau g(\tau_0,\tau)\frac{kv_B^{(1)}(k,\tau)}{\dot{\kappa}(\tau)}\frac{j_l(k(\tau_0-\tau))}{k(\tau_0-\tau)} \right|^2 > . \quad (2)$$

Here we have included an extra factor of 2 to account for the fact that tangled magnetic fields induce both $m = +1$ and $m = -1$ contributions to the vector perturbations, and one has to sum over the power in both these to get the net power in B-type polarization anisotropy. The above expression for $C_l^{BB}$ is very closely related to the corresponding expression for temperature power spectrum $C_l$, due to tangled magnetic fields given in Eq. (1) of Ref. [9] (henceforth, Paper I), where KS was a co-author. One can make the same approximations as made there, to obtain an analytic estimate of $C_l^{BB}$. We nevertheless repeat the arguments here, for clarity.

To begin with, it suffices to approximate the visibility function as a Gaussian: $g(\tau_0, \tau') = (2\pi\sigma^2)^{-1/2}\exp[-(\tau'-\tau_*)^2/(2\sigma^2)]$, where $\tau_*$ is the conformal epoch of "last scattering" and $\sigma$ measures the width of the last scattering surface (LSS). Using the expressions given in Ref. [12], we estimate $\tau_* \sim 178.2h^{-1}Mpc$ and $\sigma = 11.1h^{-1}Mpc$. ($h$ is the Hubble constant in units of 100 km s$^{-1}$ Mpc$^{-1}$.) Due to the visibility function, the dominant contributions to the integral over $\tau$ in Eq. (2) come from conformal times in a range $\sigma$ around the epoch $\tau = \tau_*$. Further the presence of $j_l(k(\tau_0-\tau))$ picks out $(k,\tau)$ values in the integrand which have $k(\tau_0-\tau) \sim l$.

Now consider wavelengths such that $k\sigma << 1$. In this case $k(\tau_0-\tau)$, and hence $j_l(k(\tau_0-\tau))$, vary negligibly over the range of $\tau$ where $g$ is significant. So they can be evaluated at $\tau = \tau_*$ and taken out of the integral over $\tau$ in Eq. (2). Also, in general, $kL_\gamma v_B^{(1)}(k,\tau)$ does not vary rapidly with conformal time within $\sigma$, nor does it vary rapidly with $k$ in the $k$ range around $k \sim l/R_*$ where $j_l(kR_*)$ contributes dominantly (we define $R_* = \tau_0 - \tau_*$). So it can also be evaluated at $\tau = \tau_*$ and $k = l/R_*$ and pulled out of the integrals. The remaining integral of $g$ over $\tau$ gives unity, while that over $j_l^2$ can be done analytically, giving for $l \gg 1$,

$$\frac{l(l+1)C_l^{BB}}{2\pi} \approx \left(\frac{kL_\gamma(\tau_*)}{3}\right)^2 \frac{\pi}{4}\Delta_v^2(k,\tau_*)|_{k=l/R_*}. \quad (3)$$

Here, $\Delta_v^2(k,\tau_*) = k^3 < |v_B^{(1)}(k,\tau_*)|^2 + |v_B^{(-1)}(k,\tau_*)|^2 > /(2\pi^2)$ is the power per unit logarithmic interval of $k$, residing in the *net* rotational velocity perturbation.

In the other limit, $k\sigma >> 1$, for wavelengths much smaller than the thickness of the LSS, $g$ can be treated as a slowly-varying function compared to $j_l$ in the integral over $\tau$ in (2). The oscillating contributions of $j_l$ over the thickness of the LSS, in the $\tau$ integral, lead to a cancellation effect. An approximate evaluation of the angular power spectrum in this case gives

$$\frac{l(l+1)C_l^{BB}}{2\pi} \approx \left(\frac{kL_\gamma(\tau_*)}{3}\right)^2 \frac{\sqrt{\pi}}{4}\frac{\Delta_v^2(k,\tau_*)}{k\sigma}|_{k=l/R_*}. \quad (4)$$

In this small-wavelength case, the angular power-spectrum is suppressed by a $1/k\sigma$ factor due to the finite thickness of the LSS. (The damping effects of radiative viscosity are incorporated when solving for the evolution of $v_B^{(1)}$ via eqn. (5) below.)

To evaluate $C_l^{BB}$, one needs to estimate the rotational component of the baryon fluid velocity field, for a general spectrum of magnetic inhomogeneities. We assume the magnetic field to be initially a Gaussian random field. On galactic scales and above, the velocity induced on the baryons is generally so small that it does not lead to any appreciable distortion of the initial field [6]. So, to a very good approximation, the evolution of the magnetic field is simply a dilution by the Hubble expansion, $\mathbf{B}(\mathbf{x},t) = \mathbf{b}_0(\mathbf{x})/a^2$. The Lorentz force associated with the tangled field is then $\mathbf{F}_L = (\nabla\times\mathbf{b}_0)\times\mathbf{b}_0/(4\pi a^5)$, which pushes the fluid, creating rotational velocity perturbations. Further, the stresses associated with the tangled magnetic field, say $\Pi_B$, can lead to metric perturbations. We focus on the perturbations with co-moving length scales larger than the photon mean-free-path at decoupling, and describe the viscous effect due to photons, in the diffusion approximation. The Fourier transform of the linearised Euler equation for the rotational perturbations in the baryon-photon fluid is given by [11,6],

$$\left(\frac{4}{3}\rho_\gamma + \rho_b\right)\frac{\partial(v_B^{(1)} - V)}{\partial t} + \frac{\rho_b}{a}\frac{da}{dt}(v_B^{(1)} - V) + \frac{k^2\eta}{a^2}v_B^{(1)}$$
$$= \frac{F_B^{(1)}}{4\pi a^5}. \quad (5)$$

Here as before, $v_B^{(1)}(k,t)$ is the magnitude of the rotational component of the velocity in Fourier space, while $V(k,t)$ is the vector component of the metric perturbation ($t$ is comoving proper time). The photon and baryon densities are given by $\rho_\gamma$ and $\rho_b$ respectively. Also $\eta = (4/15)\rho_\gamma l_\gamma$ is the shear viscosity coefficient associated with the damping due to photons, whose mean-free-path is $l_\gamma = L_\gamma a(t)$. $F_B^{(1)}$ is the rotational part of the Lorentz force, defined through the relation $F_B^{(1)}Q_i^{(1)} = P_{ij}F_j$, where the vector $\mathbf{Q}^{(\pm 1)} = -i(\mathbf{e_1} \pm i\mathbf{e_2})$, with the unit vectors $\mathbf{e_1}$ and $\mathbf{e_2}$ being normal to each other and the unit vector $\mathbf{e_3} = \mathbf{k}/k$. We have defined the Fourier transforms of the magnetic field as, $\mathbf{b}_0(\mathbf{x}) = \sum_{\mathbf{k}} \mathbf{b}(\mathbf{k}) \exp(i\mathbf{k}.\mathbf{x})$. Since the Lorentz force is non-linear in $\mathbf{b}_0(\mathbf{x})$, this leads to the mode-coupling term $\mathbf{F}(\mathbf{k}) = \sum_{\mathbf{p}}[\mathbf{b}(\mathbf{k}+\mathbf{p}).\mathbf{b}^*(\mathbf{p})]\mathbf{p} - [\mathbf{k}.\mathbf{b}^*(\mathbf{p})]\mathbf{b}(\mathbf{k}+\mathbf{p})$. The projection tensor, $P_{ij}(\mathbf{k}) = [\delta_{ij} - k_ik_j/k^2]$ projects $\mathbf{F}$ onto its transverse components (perpendicular to $\mathbf{k}$ ).

The comoving Silk scale at recombination, $L_S \sim (l_\gamma(t_*)t_*)^{1/2}/a(t_*)$, separates scales for which the damping term in (5), is important ($k^{-1} << L_S$) from those for which it is negligible ($k^{-1} >> L_S$). We can solve Eq.(5) analytically, in these two limits. For $kL_s < 1$, and when the fluid starts from rest ($v_B^{(1)}(\tau_i) = 0$), the damping due to the photon viscosity can be neglected compared to the driving due to the Lorentz force. Integrating gives $v_B^{(1)} = V + G_B^{(1)}(\tau - \tau_i)/(1+S_*)$, where we have defined $G_B^{(1)} = 3F_B^{(1)}/[16\pi\rho_0]$, with $\rho_0$ the redshifted present day value of $\rho_\gamma$, and $S_* = (3\rho_b/4\rho_\gamma)(\tau_*) \sim 0.4 f_b$. ($f_b \equiv (\Omega_b/0.0125h^{-2})$, with $\Omega_b$ the baryonic density parameter.) The $(1+S_*)$ factor results from the reduction in the induced velocity due to baryon inertia. One can check from this solution that the neglect of viscous damping is valid for $kL_S < 1$. One can also show that [13], the metric perturbation term $V$ is smaller than the Lorentz force driven contribution to $v_B^{(1)}$, for large $l$ by a factor $\sim (l/30)^{-2}h_{75}^{-2}$ ($h_{75} = (h/0.75)$); and so makes a negligible contribution for the small angular scale ($l > 300$), polarization anisotropy that we consider here. In the other limit, with $kL_s >> 1$, we can use the terminal-velocity approximation, neglecting the inertial terms in the Euler equation, and simply balance the Lorentz force by friction. This gives $v_B^{(1)} = (G_B/k)(kL_\gamma/5)^{-1}$, independent of the metric perturbation $V$.

We also need to specify the spectrum of the tangled magnetic field, $M(k)$. We define, $< b_i(\mathbf{k})b_j(\mathbf{q}) >= \delta_{\mathbf{k},\mathbf{q}}P_{ij}(\mathbf{k})M(k)$, where $\delta_{\mathbf{k},\mathbf{q}}$ is the Kronecker delta which is non-zero only for $\mathbf{k} = \mathbf{q}$. This gives $< \mathbf{b}_0^2 >= 2\int (dk/k)\Delta_b^2(k)$, where $\Delta_b^2(k) = k^3M(k)/(2\pi^2)$ is the power per logarithmic interval in $k$ space residing in magnetic tangles, and we replace the summation over $k$ space by an integration. The ensemble average $< |v_B^{(1)}|^2 >$, and hence the $C_l^{BB}$s, can be computed in terms of the magnetic spectrum $M(k)$. It is convenient to define a dimensionless spectrum, $h(k) \equiv \Delta_b^2(k)/(B_0^2/2)$, where $B_0$ is a fiducial constant magnetic field. We will also define the Alfvén velocity, $V_A$, for this fiducial field, using

$$V_A = \frac{B_0}{(4\pi\rho_0/3))^{1/2}} \approx 3.8 \times 10^{-4} B_{-9}. \qquad (6)$$

where we have expressed $B_0$ the present day field strength, in units of $10^{-9}$ Gauss. Also, as a measure of the B-type CMBR polarization anisotropy induced by the tangled magnetic field, we define the quantity $\Delta T_P^{BB}(l) \equiv [l(l+1)C_l^{BB}/2\pi]^{1/2}T_0$.

Since scales with $kL_s < 1$ also generally satisfy the criterion $k\sigma < 1$, the resulting CMBR anisotropy on these scales can be estimated using Eq.(3). A lengthy calculation gives, for scales with $kL_s < 1$ and $k\sigma < 1$,

$$\Delta T_P^{BB}(l) = T_0(\frac{\pi}{32})^{1/2} I(k) \frac{kV_A^2\tau_*}{(1+S_*)} \frac{kL_\gamma(\tau_*)}{3}$$
$$\approx 0.23\mu K \left(\frac{B_{-9}}{3}\right)^2 \left(\frac{l}{300}\right)^2 I(\frac{l}{R_*})\frac{h_{75}}{f_b} \qquad (7)$$

Here, $l = kR_*$, $S_* = 0.4$, and we have assumed $\tau_* >> \tau_i$.

For scales with $kL_S > 1$ and $k\sigma > 1$, we can use Eq.(4), and $v_B^{(1)} = (G_B/k)(kL_\gamma/5)^{-1}$. A similar calculation to that above gives

$$\Delta T_P^{BB}(l) = T_0 \frac{\pi^{1/4}}{\sqrt{32}} I(k) \frac{5V_A^2}{3(k\sigma)^{1/2}}$$
$$\approx 0.82\mu K \left(\frac{B_{-9}}{3}\right)^2 \left(\frac{l}{1500}\right)^{-1/2} I(\frac{l}{R_*}). \qquad (8)$$

Note that $kL_\gamma$ dependence in Eq.(8) has cancelled out as the induced velocity is $\propto (kL_\gamma)^{-1}$ in the terminal velocity limit, and the polarization is proportinal to $kL_\gamma$ times the velocity in the tight-coupling limit. We also note that in both cases, the polarization anisotropy, $\Delta T_P^{BB}(l) \approx (kL_\gamma(\tau_*)/3) \times \Delta T(l)$, where, $\Delta T(l)$ is the temperature anisotropy computed in Paper I. Since we are considering scales with $kL_\gamma < 1$, (and small departures from tight-coupling), the polarization anisotropy ($\sim 0.5\mu K$), is much smaller than the temparature anisotropy due to tangled magnetic fields ($\sim 5\mu K$), as it should be!

The function $I^2(k)$ in the eqs.(7)-(8) is a dimensionless mode-coupling integral given as in Paper I by

$$I^2(k) = \int_0^\infty \frac{dq}{q} \int_{-1}^{1} d\mu \frac{h(q)h(|(\mathbf{k}+\mathbf{q})|)k^3}{(k^2+q^2+2kq\mu)^{3/2}}$$
$$\times (1-\mu^2)\left[1 + \frac{(k+2q\mu)(k+q\mu)}{(k^2+q^2+2kq\mu)}\right] \qquad (9)$$

where $|(\mathbf{k}+\mathbf{q})| = (k^2+q^2+2kq\mu)^{1/2}$. In the simple case when the magnetic spectrum has a single scale, with $h(k) = k\delta_D(k-k_0)$, where $\delta_D(x)$ is the Dirac delta function, $< \mathbf{b}_0^2 >= B_0^2$ and the mode-coupling integral can be evaluated exactly. We find $I(k) = (k/k_0)[1 - (k/2k_0)^2]^{1/2}$, for $k < 2k_0$, and $I(k) = 0$ for larger $k$. So $I(k)$ contributes a factor of order unity near $k \sim k_0$, with $I(k_0) = \sqrt{3}/2$. For more complicated magnetic spectra, with a multitude of scales, $I(k)$ can be thought of as a superposition of these elementary contributions. It could be somewhat larger, but has to be numerically evaluated. Also for a power law spectra with $M(k) \propto k^n$, with appropriate cut-offs at small and large $k$'s, we will have a power-law regime where $I(k) \propto k^{3+n} \propto l^{3+n}$.

Earlier work also emphasised the possibility of Faraday rotation and the depolarization of the CMBR due to differential Faraday rotation in a tangled magnetic field [14]. Note that the average Faraday rotation (in radians) between Thomson scatterings is given by $F = 3B_0/(2\pi e\nu_0^2) \approx 0.23(B_0/3\times10^{-9}G)(\nu_0/30GHz)^{-2}$, where $\nu_0$ is the observed frequency. As pointed out in by Harari *et al.* in Ref. [14] the CMB could become significantly de-polarised due to this effect, for $\nu_0 < 16.4GHz(B_0/3\times10^{-9}G)^{1/2}$. So the effect, while important at "low-frequenies" is likely to be negligible for say $\nu_0 > 40GHz$, or the higher frequency instruments of the Planck Satellite.

Note also that while scalar perturbations, due to tangled magnetic fields do not contribute B-type polarization, they could give an E-type contribution. However, this is sourced by compressional fluid perturbations, which are of small amplitude ($\sim V_A^2 << 1$), compared to rotational perturbations, because of the larger restoring force contributed by the pressure of the radiation-baryon fluid [6]. They are also strongly damped on scales smaller than the Silk scale, $L_S$, while the Alfvén mode, vector perturbations survive damping on much smaller comoving scales $> L_A \sim V_A L_S$ [6]. Tensor metric perturbations can also contribute to the polarization anisotropy, but one can show that their effect at small-angular scales is much smaller than the vector contribution driven by the Lorentz-force. A more detailed computation of these effects will be presented elsewhere.

In summary, from Eqs.(7)-(8), we see that for a tangled field of order $B_0 \sim 3\times10^{-9}G$, one expects a RMS B-type CMBR polarization anisotropy of order $0.2\mu K - 0.8\mu K$ or larger, depending on the contribution of $I(k)$ and the value of $l$. The anisotropy in hot or cold spots could be several times larger, because the non-linear dependence of $C_l^{BB}$ on $M(k)$ will imply a non-Gaussian statistics for the anisotropies (see Paper I). Further in standard models all the $C_l s$ have a sharp cut-off for $l > R_*/L_S$, due to Silk damping but strong damping of Alfvénic perturbations is expected only on scales smaller than $V_A L_S$ [6]. The damping due to the finite LSS thickness, at small scales, is also milder for vector modes. Finally, since the polarization arising due tangled magnetic fields are predominantly of B-type, one can distinguish these signals from those produced by scalar perturbations.

We have identified in this work, a new physical effect of tangled magnetic fields; that they can produce distinctive and potentially detectable B-type polarization anisotropy on arc minute scales. Satellite bourne experiments like Planck are indeed expected to be able to map the polarization anisotropy of the CMBR at the levels predicted here. They should be able to detect and isolate the effects of magnetic fields, using CMBR polarization, if such fields indeed play a role in structure formation.

As this letter was prepared for submission, a preprint [15] of a conference paper appeared, which has some overlap with the present paper.


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[13] Note that $V$ satisfies the vector Einstien equation, $\dot{V} + 2(\dot{a}/a)V = -8\pi G a^2 \Pi_B^{(1)}/k$, where 'dot' denotes a conformal derivative (see Eq (70) of Ref. ( [11])). and the vector part of the stress, $\Pi_B^{(1)} = F_B^{(1)}/k$. This integrates to give $V = -8\pi G F_B^{(1)}(\tau - \tau_i)/(k^2 a^2)$. The ratio of $V$ to the Lorentz force term at last scattering is then $-8\pi G(4\rho_0/3)(1+S_*)/(k^2 a(\tau_*)^2) \sim (l/30)^{-2} h_{75}^{-2}$. So we can neglect the $V$ term contribution for large $l > 100$.

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